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Energy-shifting formulae yield reliable reaction and capture probabilities

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ABSTRACT

Predictions of energy-shifting formulae for partial reaction and capture probabilities are compared with coupled channels calculations. The quality of the agreement notably improves with increasing mass of the system and/or decreasing mass asymmetry in the heavy-ion collision. The formulae are reliable and useful for circumventing impracticable reaction calculations at low energies.

1. Introduction

The physics of low-energy nuclear reactions is critical for understanding energy production and nucleosynthesis in the universe [1]. The heavy-ion collisions at energies near the Coulomb barrier are highly affected by the interplay of nuclear structure and reaction dynamics [2–4], the bare Coulomb barrier being modified by couplings and centrifugal effects. Coupled-reaction-channels (CRC) calculations provide partial, transmission and reflection coefficients that determine a number of reaction observables such as capture and reaction cross sections. Although feasible nowadays within various implementations [5,6], the CRC calculations can be sometimes computationally demanding and time consuming [7], so a simple formula for the partial transmission and reflection coefficients seems to be useful. The energy-shifting formula has a long history across disciplines [8,9]. The present paper addresses the quality of energy-shifting formulae for partial reaction and capture probabilities at near-barrier energies. We first present the energy-shifting formulae and a description of the CRC calculations, followed by results and a summary.

2. Energy-shifting formulae

The key idea of the energy-shifting formula [8–10] consists in replacing the exact, reaction and capture probabilities for a nonzero partial wave J and a given incident energy E, $P_{reac}(E, J)$ and $P_{cap}(E, J)$, with the corresponding s-wave probabilities evaluated at a lower energy:

$$P_i(E, J) \approx P_i(\epsilon_J, J = 0), \tag{1}$$

where $i = \{reac, cap\}$ and $\epsilon_J = E - E_{rot}(J)$, $E_{rot}(J)$ being interpreted as a rotational energy. In the present paper, $E_{rot}(J)$ is calculated in two different ways:

(i) The first way relies on expanding the Coulomb barrier height, $V_B(J)$, up to second order in $\Lambda = J(J+1)$, and subtracting the s-wave potential barrier, $V_B(0)$, so the rotational energy reads as [8]:

$$E_{rot}(J) = \frac{\hbar^2 \Lambda}{2\mu R_B^2} + \frac{\hbar^4 \Lambda^2}{2\mu^3 \omega_B^2 R_B^6}, \tag{2}$$

where μ denotes the reduced mass of the projectile–target radial motion, R_B and ω_B are the radius and curvature of the s-wave barrier, respectively. Using the lowest order in Eq. (2), an energy-shifting formula has been used for deriving the well-known Wong formula [11,12]. The second order correction term in Eq. (2) takes into account the dependence of the barrier radius on the angular momentum.

(ii) The second way is based on a nuclear-modified Rutherford trajectory for the near-barrier projectile–target orbit [13], $E_{rot}(J)$ being identified as the rotational energy at the distance of closest approach:

$$E_{rot}(J) = E\frac{(\eta'^2 + J^2)^{1/2} - \eta'}{(\eta'^2 + J^2)^{1/2} + \eta'}, \tag{3}$$

where $\eta' = Z'\sqrt{\mu/2\hbar^2 E}$ is an effective Sommerfeld parameter that takes into consideration the nuclear part of the nucleus–nucleus interaction potential through $Z' = Z_P Z_T e^2(1 - a_0/R_B)$

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Table 1
Parameters of both the bare Woods–Saxon potential between the colliding nuclei (second column) and the uncoupled Coulomb barriers, used in FRESCO (first three rows) and CCFULL (last two rows). Energy is in MeV, while radius and diffuseness are in fm.

Reactions	(V_0, r_0, a_0)	V_B	R_B	$\hbar\omega_B$
^{4}He + ^{120}Sn	(−34, 1.104, 0.65)	14.21	9.41	4.63
^{16}O + ^{120}Sn	(−56.7, 1.193, 0.635)	50.75	10.63	4.23
^{16}O + ^{208}Pb	(−59.5, 1.2, 0.645)	75.94	11.7	4.66
^{16}O + ^{208}Pb	(−100, 1.17, 0.66)	75.04	11.86	4.76
^{16}O + ^{154}Sm	(−165, 0.95, 1.05)	59.41	10.81	3.48

Table 2
Properties of the target nuclei used in FRESCO for constructing model-independent, coupling form-factors.

Nucleus	Energy (MeV) $(2^+, 3^-)$	Deform. length (fm) (δ_2, δ_3)	Strength (e fm^k) M(E2), M(E3)
^{120}Sn	(1.17, 2.40)	(0.64, 0.81)	(44.94, 338.02)
^{208}Pb	(4.07, 2.61)	(0.40, 0.80)	(54.45, 815.00)

[13], whereas Z_i and a_0 refer to the charge number of the projectile and target nuclei and the diffuseness parameter of their bare nuclear interaction, respectively. To obtain Eq. (3), the unique relation between the entrance-channel J and the exit-channel scattering angle in the center-of-mass system, $J = \eta' \cot(\theta/2)$, has been used. An expression like Eq. (3), without nuclear corrections, was employed in Refs. [14,15] to study quasi-elastic barrier distributions.

The exact probabilities for reaction and capture, $P_{reac}(E, J)$ and $P_{cap}(E, J)$, are determined with CRC calculations as described below. Having calculated these probabilities, the quality of Eq. (1) will be studied using (i) and (ii), and the associated formulae are called energy-shifting formulae 1 and 2. It is worth mentioning that a formula for the capture excitation function in terms of s-wave capture probabilities was proposed in Ref. [12], which stems from the eigenchannels picture.

3. CRC calculations

Coupled channels calculations have been carried out for four reactions using the FRESCO and CCFULL codes [5,6].

Table 1 (first three rows) shows the bare nuclear Woods–Saxon potential between the colliding nuclei used in FRESCO, while the Coulomb potential is that for a uniformly charged sphere with radius of $1.2(A_P^{1/3} + A_T^{1/3})$ fm. The parameters of the bare Coulomb barrier are similar to those of the Sao-Paulo potential barriers [16]. A short-range imaginary (squared Woods–Saxon) potential accounts for fusion [17], whose parameters are $(W_{0I}, r_{0I}, a_{0I}) = (-10.0\ \text{MeV}, 1.0\ \text{fm}, 0.4\ \text{fm})$. The projectile is considered inert, while in the target nucleus, beside its ground-state, one-phonon 2^+ and 3^- excitations are included in the coupling scheme that is model-independent. Table 2 presents observed properties of the target nuclei [18,19], which are used for constructing the FRESCO coupling form-factors [5]. The FRESCO code solves the CRC equations with partial waves up to $J = 300$ and asymptotic Coulomb-wave boundary conditions, whose outcomes include the elastic S-matrix elements, $S_J(E)$, which determine the partial reaction probabilities, $P_{reac}(E, J) = 1 - |S_J(E)|^2$.

Partial capture probabilities, $P_{cap}(E, J)$, have been studied with the CCFULL code [6]. This code is based on the iso-centrifugal approximation which works well in low-energy heavy-ion fusion [4]. The nuclear coupling which has a deformed Woods–Saxon form is treated to all orders, while the Coulomb coupling includes terms up to second order with respect to the quadrupole deformation parameter and to first order beyond the quadrupole one [6]. In the calculations below, the deformation parameters of the nuclear and Coulomb coupling potentials are considered the same, the vibrational coupling being in the harmonic limit. The transmission coefficient through the Coulomb barrier is determined by ingoing-wave boundary condition at the minimum of the pocket in the entrance channel potential, which is equivalent to a short-range absorption around the potential pocket [4]. Table 1 (last two rows) displays the bare Woods–Saxon potential parameters for both the ^{16}O + ^{208}Pb and ^{16}O + ^{154}Sm collisions, while the Coulomb potential is that for two point charges. The CCFULL calculations use a coupling radius parameter of 1.06 fm and include only the target excitations: (i) for ^{208}Pb, two-phonon 3^- ($E_{3^-} = 2.62$ MeV with $\beta_3 = 0.161$) and one-phonon 5^- ($E_{5^-} = 3.2$ MeV with $\beta_5 = 0.056$), and (ii) for ^{154}Sm, couplings up to the 8^+ state of the ground-state rotational band with $E_{2^+} = 0.08$ MeV ($\beta_2 = 0.322$) and $E_{4^+} = 0.27$ MeV ($\beta_4 = 0.027$), respectively.

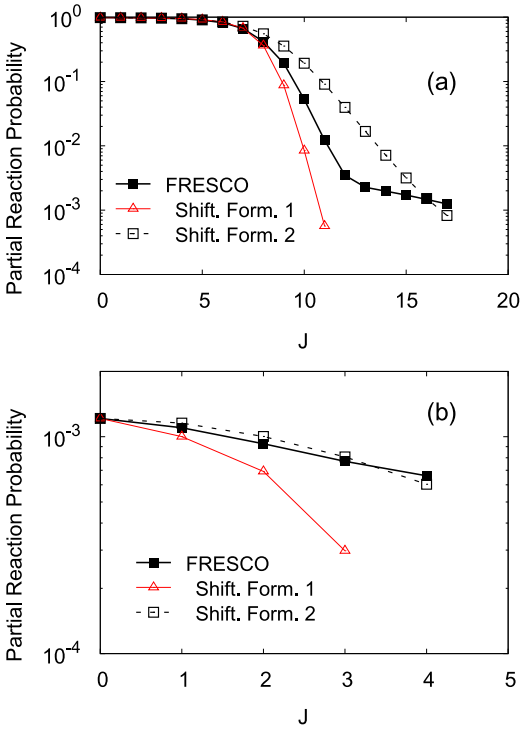

Fig. 1. Partial reaction probability for ^{4}He + ^{120}Sn at two center-of-mass energies around the Coulomb barrier: (a) $E = 18.39$ MeV and (b) $E = 10.16$ MeV. The FRESCO results are sandwiched by those of the two energy-shifting formulae.

4. Results

Fig. 1 shows the partial reaction probability for ^{4}He + ^{120}Sn for two incident energies, (a) above and (b) below the s-wave Coulomb barrier. The FRESCO results are between the two energy-shifting formulae, the agreement becoming better at sub-Coulomb energy [Fig. 1(b)]. The formula results seem very sensitive to $E_{rot}(J)$, Eqs. (2) and (3), as J increases. This sensitivity declines with a heavier projectile like ^{16}O (Fig. 2) or a heavier system like ^{16}O + ^{208}Pb (Fig. 3). Figs. 2 and 3 also show that the FRESCO results are sandwiched by the formula outcomes. The same features are revealed in FRESCO calculations within the iso-centrifugal approximation.

It appears that the quality of the agreement between the FRESCO calculations and the formula results improves notably, as the *total mass* $A = A_T + A_P$ increases and/or the *mass asymmetry* $[\eta = (A_T - A_P)/A]$ decreases, from ^{4}He + ^{120}Sn ($\eta = 0.93$),

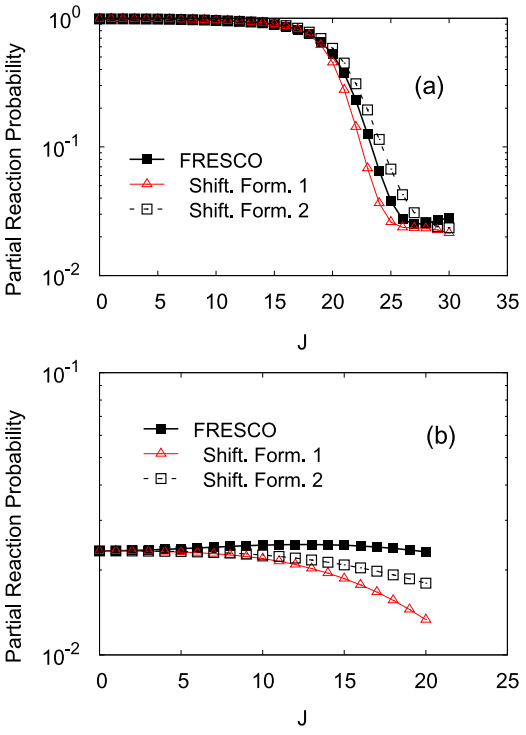

Fig. 2. The same as in Fig. 1, but for $^{16}O + ^{120}Sn$: (a) $E = 56.76$ MeV and (b) $E = 45$ MeV.

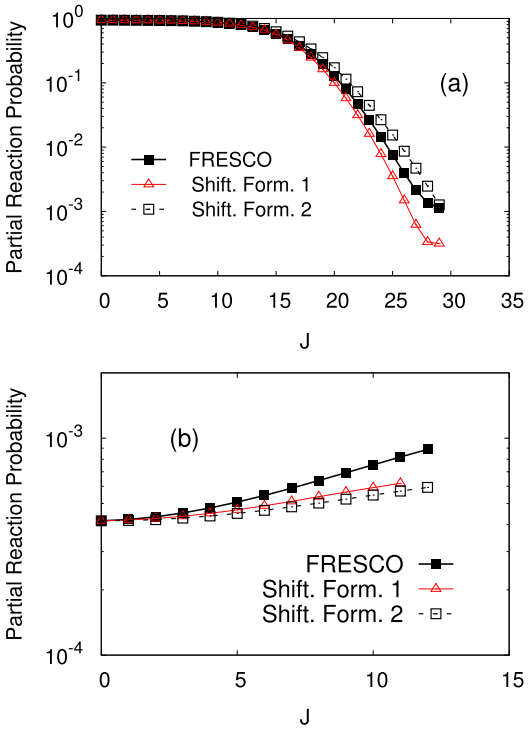

Fig. 3. The same as in Fig. 1, but for $^{16}O + ^{208}Pb$: (a) $E = 78$ MeV and (b) $E = 68.25$ MeV.

$^{16}O + ^{208}Pb$ ($\eta = 0.86$) to $^{16}O + ^{120}Sn$ ($\eta = 0.76$). These two variables play an important role in the quality of the agreement as they impact on the moment of inertia of the dinuclear system. For instance, considering the moment of inertia $I = \mu R_B^2$, where $\mu \sim A(1-\eta^2)/4$ and $R_B \sim A^{1/3}[(1-\eta)^{1/3} + (1+\eta)^{1/3}]$, the moment of inertia will be $I \sim A^{5/3}(1-\eta^2)[(1-\eta)^{1/3} + (1+\eta)^{1/3}]^2$.

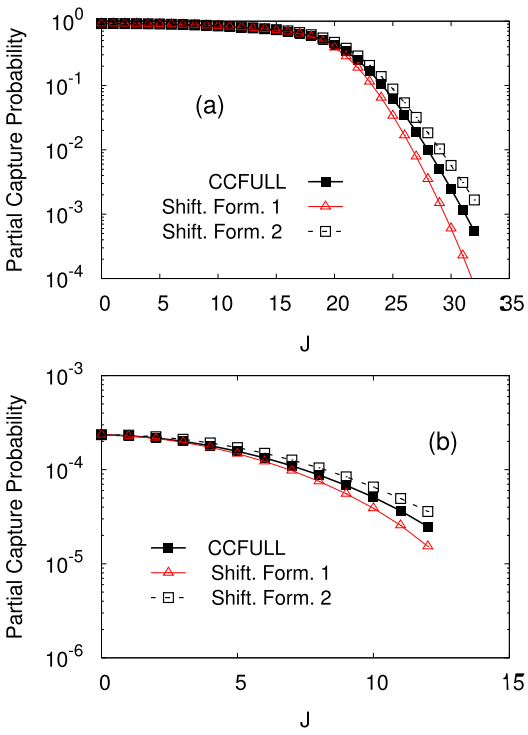

Fig. 4. Partial capture probability for $^{16}O + ^{208}Pb$ at two near-barrier energies: (a) $E = 78$ MeV and (b) $E = 68$ MeV.

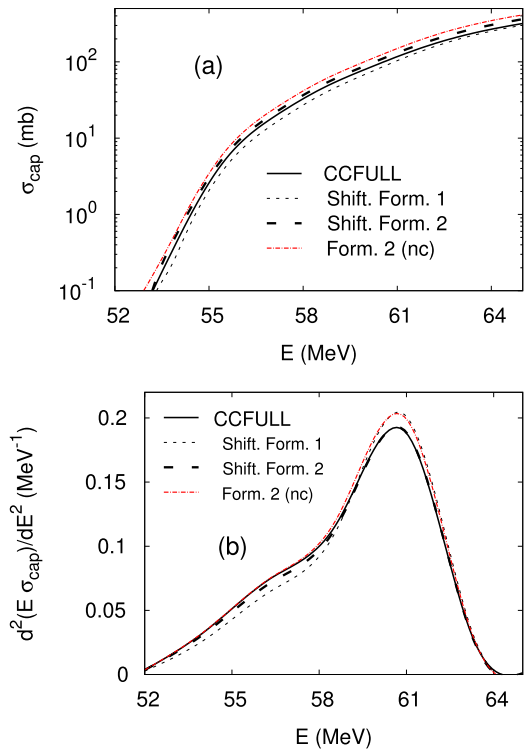

Fig. 5. (a) Capture excitation function and (b) capture barrier distribution for $^{16}O + ^{154}Sm$. The exact, CCFULL outcomes are compared to those resulting from the energy-shifting formulae, the barrier-distribution areas being normalized to unity. The three-point finite-difference formula with $\Delta E = 1.8$ MeV has been used for the second derivative [15,20]. The agreement confirms the quality of the energy-shifting formulae, which is better for formula 2 including the nuclear correction in Eq. (3).

Consequently, I rises with increasing A and decreasing η, and the role of the centrifugal potential declines. It reduces the increasing rate of the rotational energy $E_{rot}(J)$ as J changes. Also the difference between R_B and the distance of closest approach becomes smaller, so Eqs. (2) and (3) provide similar rotational energies. Small J values ($J \leq 30$–40) are the main contributors to the heavy-ion capture and reaction cross sections at energies around the Coulomb barrier. The energy-shifting formulae are reliable in these collisions.

Fig. 4 presents partial capture probabilities for $^{16}O + ^{208}Pb$ at two energies around the Coulomb barrier. These calculations use the CCFULL capture probabilities. It can be seen that the energy-shifting formulae also perform adequately, sandwiching the CCFULL outcomes. The quality of the formulae is confirmed by the good agreement between both the exact, capture excitation function and associated barrier distribution and those of the energy-shifting formulae, as shown in Fig. 5 for $^{16}O + ^{154}Sm$. The nuclear correction in Eq. (3) appears to be essential for the excellent performance of formula 2 in Fig. 5 (comparing the dashed line with the dash-dotted line that does not include this correction).

5. Summary

Comparing the predictions of the energy-shifting formulae for partial reaction and capture probabilities with those of direct coupled channels calculations, it appears that the quality of the agreement remarkably growths with increasing mass of the dinuclear system and/or decreasing mass asymmetry in the heavy-ion collision. The formulae are reliable and useful for simplifying (or avoiding impracticable) calculations of heavy-ion reaction observables at low energies.

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